

Practical Operation Strategy for Deorbit of an Electrodynamic Tethered System

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DOI: 10.2514/1.19635

This paper presents an operation strategy employing electric current switching for deorbit of an electrodynamic tethered system. The electric current in the tether experiences some unexpected changes due to its plasma environment and thermal conditions. Libration control through current switching, in which only the on/off status of the current is determined, can avoid such changes and unexpected instability. A simple control law to determine current switching to stabilize both in-plane and out-of-plane libration is presented. A new stability function is also presented to facilitate evaluating the stability of a tethered system on an elliptic orbit. The effectiveness of the libration control strategy and the stability function is demonstrated through several numerical simulations. It is clearly indicated that the stability function and a switching control toward a periodic solution facilitate an appropriate stability evaluation and an efficient libration control. The control strategy is considered to be easily materialized in actual electrodynamic tethered systems, and enables a more rapid and secure deorbit.

Nomenclature

a	= orbital semimajor axis
\mathbf{B}^o	= geomagnetic field vector in orbital coordinate
e	= orbital eccentricity
\mathbf{F}^o	= Lorentz force vector acting on tether in orbital coordinate
I	= intensity of current
i	= orbital inclination measured from geomagnetic equator
\mathbf{L}^o	= direction vector of tether in orbital coordinate
l	= length of tether
M	= mass of debris
M_r	= reduced mass of EDT; $M_r = Mm/(M + m)$
m	= tip mass of EDT
Q_θ, Q_ϕ	= generalized forces
r	= orbital radius
x^l, y^l, z^l	= inertia coordinate system
x^o, y^o, z^o	= orbital coordinate system
θ	= in-plane attitude angle
μ_m	= strength of magnetic dipole
μ_g	= Earth gravitational constant
v	= orbital true anomaly
ϕ	= out-of-plane attitude angle
ψ	= change angle of orbital plane

Introduction

RECENTLY, the increase of space debris on Earth orbits is becoming a threat to future space activities [1]. First, malfunctioning or end-of-life satellites and upper stages of spent rockets turn into large-scale debris. Next, the explosion of the remaining propellant and collisions between large debris generate considerable small debris. The debris is thus expected to increase significantly in the near future. To maintain the space environment for sustainable space activities, research and development has been conducted to realize the debris removal system [2]. The

electrodynamic tethered system [3,4] (EDT), shown in Fig. 1, is an important technology that enables orbital transfer by using electromagnetic force. An EDT uses a current-conductive tether and has electronic equipment to exchange electrons with the neighboring plasma environment. It uses the Lorentz force generated between the tether current and the geomagnetic field for its orbital transfer. In this way, an EDT facilitates an extremely efficient orbital transfer without consuming any propellant.

Previous studies on the dynamics of EDTs have revealed that the libration of an EDT becomes inherently unstable during its orbital descent [5–7]. The amplitude of libration gradually increases in both in-plane and out-of-plane directions, the tether loses its tension, and finally, the EDT begins to tumble. Several libration controls have been presented to stabilize the libration [8–12]. In these studies, the libration control strategies using the Lorentz forces are presented because these strategies do not require additional equipment. Because the amount of the tether current depends on its plasma environment, the EDT cannot always have a sufficient amount of electrons for its desired control. Therefore, the switching control to determine on/off status of the tether current [8,9] is more appropriate for the libration control because it does not require precise current control.

This paper aims to present a practical operation strategy that is capable of controlling both in-plane and out-of-plane libration, and evaluating the stability accounting for the changes of the orbital parameters. It is expected that the control strategy will enable a rapid and secure orbital descent.

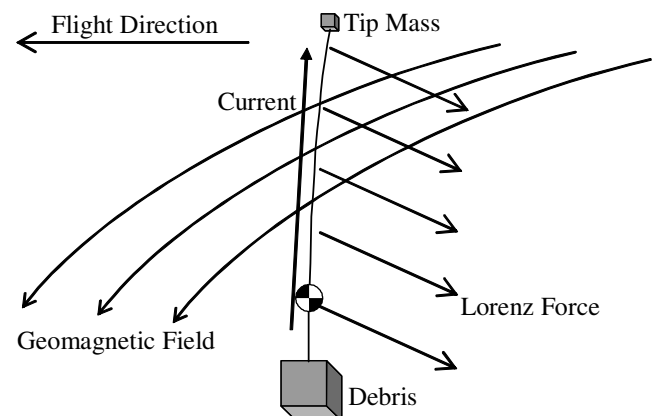


Fig. 1 Concept of electrodynamic tethered system during orbital descent.

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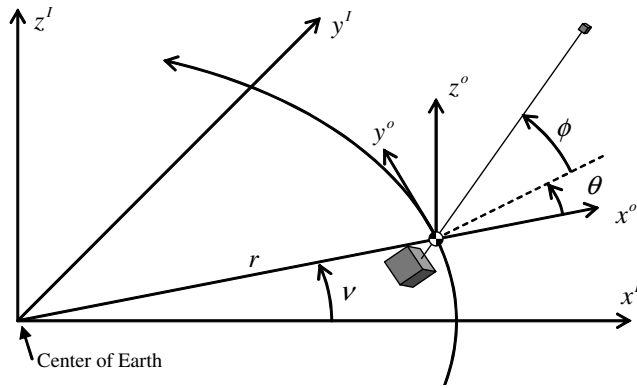


Fig. 2 Coordinates of EDT.

Mechanical Model of EDT

The coordinates of the EDT being considered are shown in Fig. 2, where the inertia coordinate system is denoted as x^I , y^I , and z^I , and the orbital coordinate system is denoted as x^o , y^o , and z^o . The EDT is modeled as two masses connected by a massless tether. Its attitude is defined by θ and ϕ . The equations of motion are given as [5]

$$\ddot{\theta} = -3\frac{\mu_g}{r^3}\cos\theta\sin\theta + 2\dot{\phi}\tan\phi(\dot{\nu} + \dot{\theta}) - \ddot{\nu} + \frac{Q_\theta}{M_r l^2 \cos^2\phi} \quad (1)$$

$$\ddot{\phi} = -3\frac{\mu_g}{r^3}\cos^2\theta\cos\phi\sin\phi - \cos\phi\sin\phi(\dot{\nu} + \dot{\theta})^2 + \frac{Q_\phi}{M_r l^2} \quad (2)$$

The Lorentz force acting on the tether is given as

$$\mathbf{F}^o = I\mathbf{L}^o \times \mathbf{B}^o \quad (3)$$

The direction vector of the tether and the geomagnetic field vector in the orbital coordinate are given as [7]

$$\mathbf{L}^o = l \begin{pmatrix} \cos\phi\cos\theta \\ \cos\phi\sin\theta \\ \sin\phi \end{pmatrix} \quad (4)$$

$$\mathbf{B}^o = \frac{\mu_m}{r^3} \begin{pmatrix} -2\sin i \sin \nu \\ \sin i \cos \nu \\ \cos i \end{pmatrix} \quad (5)$$

The torque acting on the tether due to Lorentz force is given as

$$\mathbf{M}^o = \frac{M-m}{2(M+m)} \mathbf{L}^o \times \mathbf{F}^o \quad (6)$$

Q_θ and Q_ϕ are given from the principle of virtual work as

$$\delta W = \mathbf{M}^o \cdot \mathbf{w}^o \delta t = Q_\theta \delta\theta + Q_\phi \delta\phi \quad (7)$$

$$Q_\theta = -\frac{\mu_m I l^2 (M-m)}{2(M+m)r^3} \cos\phi [\cos i \cos\phi + \sin i \sin\phi (2\cos\theta \sin\nu - \cos\nu \sin\theta)] \quad (8)$$

$$Q_\phi = \frac{\mu_m I l^2 (M-m)}{2(M+m)r^3} \sin i (\cos\nu \cos\theta + 2\sin\nu \sin\theta) \quad (9)$$

The orbital motion subjected to the Lorentz force is described as

$$\ddot{r} = -\frac{\mu_g}{r^2} + r\dot{\nu}^2 + \frac{F_x^o}{M+m} \quad (10)$$

$$\ddot{\nu} = -\frac{2\dot{r}\dot{\nu}}{r} + \frac{F_y^o}{r(M+m)} \quad (11)$$

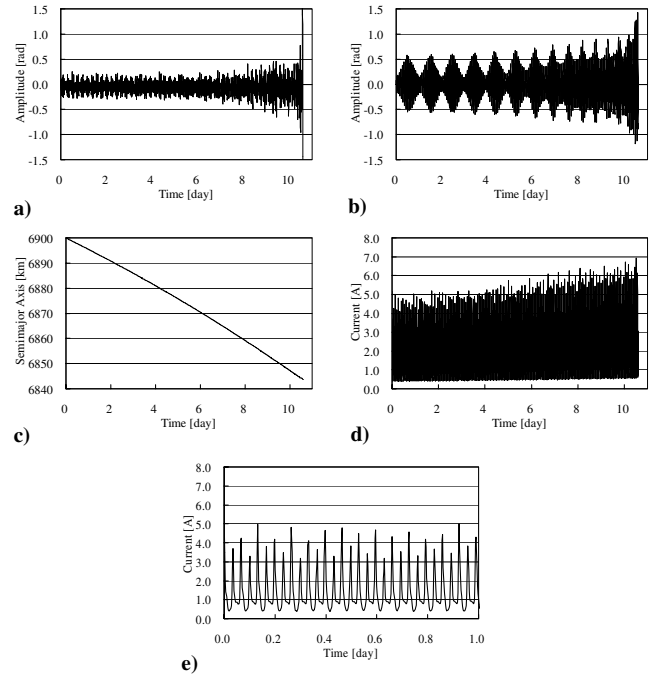


Fig. 3 Uncontrolled motion of EDT, a) In-plane libration, b) out-of-plane libration, c) semimajor axis, d) electric current, e) close-up of electric current.

$$\ddot{\psi} = -\dot{\nu}^2 \psi - \frac{2\dot{r}\dot{\psi}}{r} + \frac{F_z^o}{r(M+m)} \quad (12)$$

where ψ is the change of the orbital plane, and $\psi \ll 1$ is assumed. It is assumed that the tether current is proportional to the electron density and the square root of the induced voltage of the tether [13]. IRI2001[†] is used in the numerical simulations for the electron density.

Uncontrolled Motion of EDT

As described in many previous studies, the libration of an EDT becomes unstable during its orbital descent. The mechanical model used in this study also describes this unstable motion. A numerical simulation is conducted using the following parameters: orbital eccentricity $e = 0$, orbital semimajor axis $a = 6900$ km, $i = 60$ deg, $M = 3000$ kg, $m = 50$ kg, and $l = 5$ km. The result is displayed in Fig. 3. In this case, the EDT begins tumbling in 11 days, and the semimajor axis decreased only about 50 km.

When the orbit has some inclination, the out-of-plane libration is excited by the out-of-plane Lorentz force. The out-of-plane libration excites the in-plane libration, and the mechanical energy moves from the out-of-plane to the in-plane libration. The energy decrease of out-of-plane libration is soon resupplied by the out-of-plane Lorentz force. Therefore, one-way transfer of mechanical energy is generated, and the libration becomes unstable. The instability arises from the out-of-plane libration, and the instability appears in the in-plane libration. Therefore, it is considered that it is more efficient to suppress both in-plane and out-of-plane libration simultaneously than to suppress only in-plane libration.

Control Strategy

Basic Concept of Switching Control

From the control viewpoint, the dynamics of an EDT subjected to Lorentz force should be regarded as a tethered system subjected to an unpredictable disturbance. In this case, it is appropriate to regard the steady solution of a nonconductive tether as the control objective of an EDT libration. On a circular orbit, the steady solution is equal to

[†]Bilitza, D., International Reference Ionosphere [online database], <http://modelweb.gsfc.nasa.gov/ionos/iri.html> [cited 3 March 2005].

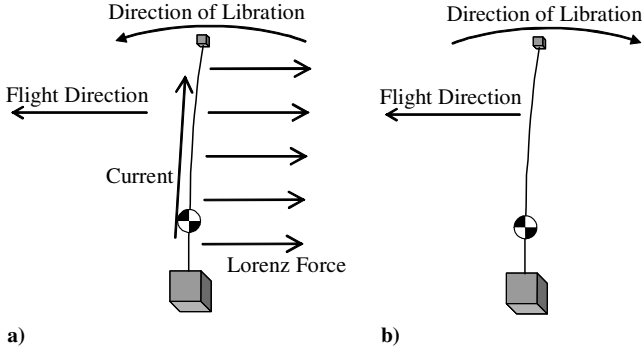


Fig. 4 Direction of Lorentz force, in-plane, a) current on, b) current off.

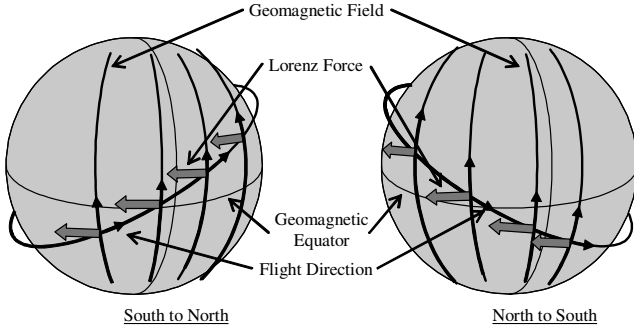


Fig. 5 Direction of Lorentz force, out-of-plane.

the equilibrium state, in which a tethered system remains still toward the center of gravity of the Earth. On an elliptic orbit, the steady solution of a nonconductive tethered system is a periodic solution, which has the same period as the orbital period [14].

For in-plane libration, the Lorentz force always accelerates the in-plane libration in the opposite direction to the flight direction (Fig. 4a). Hence, the in-plane libration can be depressed through the following switching law: turn on the electric current to decelerate the libration only when it is faster than the control objective (Fig. 4a); turn the electric current off when the libration is slower than the periodic solution (Fig. 4b). This strategy could be simplified on a circular orbit as follows: turn on the electric current to decelerate the libration only when the direction of libration is the same as the flight direction; turn the electric current off when the direction of libration is opposite to the flight direction. For out-of-plane libration, the direction of the out-of-plane Lorentz force can be identified from the orbital motion. Because the direction of the geomagnetic field is from south to north, for a posigrade EDT orbit, the direction of the current is mostly from the Earth to the zenith with the resulting Lorentz force mostly from east to west. From the viewpoint of the orbital plane, the direction of the out-of-plane Lorentz force is determined by the north-south direction of the orbital motion as depicted in Fig. 5. It is possible to control the in-plane and out-of-plane libration simultaneously by turning on the electric current only when the Lorentz force simultaneously decelerates both in-plane and out-of-plane libration. This is the basic concept of the libration control through the electric current switching.

Measure Function of Stability

The principal purpose of an EDT is to accomplish the deorbit as quickly as possible, which requires turning on the electric current for as long as possible. Therefore, a function is defined to evaluate the stability to avoid unnecessary libration control to minimize the time the current is turned off. The Hamiltonian of a tethered system on a circular orbit becomes conservative [5], and it has often been applied to stability evaluation [8] and guidance [15]. Because an EDT is essentially an orbital-transfer system, its orbital parameters always vary, which must be considered in the libration control to accomplish its deorbit securely. Among the orbital elements, eccentricity is

known to affect the dynamic stability of tethered systems [14,16,17]. In this paper, a stability function that accounts for the effect of the eccentricity is newly introduced. Because the purpose of the new function is to evaluate the stability appropriately, not for guidance and control, it should maintain the mechanical basis of the Hamiltonian on a circular orbit as much as possible.

The Hamiltonian is obtained as [5]:

$$H = \frac{1}{2} M_r l^2 \left[\frac{\mu_g}{r^3} (1 - 3\cos^2\phi\cos^2\theta) + \cos^2\phi(\dot{\theta}^2 - \dot{\nu}^2) + \dot{\phi}^2 \right] \quad (13)$$

For a tethered system on a circular orbit, the following conservative Hamiltonian is obtained by introducing the orbital mean motion n :

$$H_c = \frac{1}{2} M_r l^2 [n^2 (1 - 3\cos^2\phi\cos^2\theta) + \cos^2\phi(\dot{\theta}^2 - n^2) + \dot{\phi}^2] \quad (14)$$

On elliptic orbits, the orbital radius r and the orbital angular velocity $\dot{\nu}$ always vary, and the Hamiltonian is no longer conservative. However, it is possible to obtain a conservative function of mechanical energy when the interaction between the libration and the orbital motion are considered. Furthermore, it has also been proved that the total mechanical energy becomes minimum when the motion coincides with the periodic solution [14]. Hence, it is aimed to define a stability function by modifying the Hamiltonian (14) to take advantage of its characteristics. In this modification, the function must be minimum when the libration coincides with the periodic solution. As one candidate, the following function is introduced:

$$H_e = \frac{1}{2} M_r l^2 \{ n^2 [1 - 3\cos^2\phi\cos^2(\theta - \theta_p)] + \cos^2\phi[(\dot{\theta} - \dot{\theta}_p)^2 - n^2] + \dot{\phi}^2 \} \quad (15)$$

where the approximate periodic solution θ_p and ϕ_p are given as [14]:

$$\theta_p = e \sin \nu - \frac{1}{2} e^2 \sin 2\nu \quad (16)$$

$$\dot{\theta}_p = n(e \cos \nu - e^2 \cos 2\nu) \quad (17)$$

$$\phi_p, \dot{\phi}_p = 0 \quad (18)$$

From Eq. (15), the nondimensional form of the stability function is defined as

$$V_e = 4 - 3\cos^2\phi\cos^2(\theta - \theta_p) + \frac{1}{n^2} \{ \cos^2\phi[(\dot{\theta} - \dot{\theta}_p)^2 - n^2] + \dot{\phi}^2 \} \quad (19)$$

Because Eq. (19) does not include the changes of r and $\dot{\nu}$ of elliptic orbits, V_e cannot evaluate the stability exactly. For more secure operation, any improper evaluation must be avoided, and the value of the stability function must not be improperly smaller than that of the exact stability. Therefore, the orbital angular velocity at the apogee $n_{a=r_{apo}}$ should be used in substituting for the mean motion n so that the value is absolutely larger than that of the exact function. Hence, the stability function V_e is redefined as

$$V_e = 4 - 3\cos^2\phi\cos^2(\theta - \theta_p) + \frac{1}{n_{a=r_{apo}}^2} \{ \cos^2\phi[(\dot{\theta} - \dot{\theta}_p)^2 - n_{a=r_{apo}}^2] + \dot{\phi}^2 \} \quad (20)$$

For comparison, the following stability function V_c is also defined and evaluates the stability assuming that the EDT is on a circular orbit:

$$V_c = 4 - 3\cos^2\phi\cos^2\theta + \frac{1}{n^2} [\cos^2\phi(\dot{\theta}^2 - n^2) + \dot{\phi}^2] \quad (21)$$

The stability function V_e becomes V_c when $e \rightarrow 0$.

For libration control, threshold V_{th} is introduced. When $V < V_{th}$, the libration is evaluated to be in the stable region, and the current is kept turned on. When $V > V_{th}$, the libration control is conducted until $V < V_{th}$. In this way, libration control is applied only when necessary, and the time for libration control will be minimized.

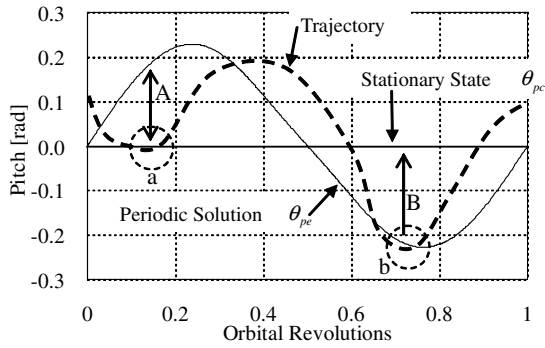


Fig. 6 Example illustration of trajectory and solutions.

Physical Interpretation of Stability Function

A more appropriate stability evaluation becomes possible by introducing a stability function accounting for the eccentricity. Figure 6 introduces an example to clearly illustrate the advantage of the newly defined function. In Fig. 6, a periodic solution of in-plane libration on an elliptic orbit is denoted as θ_{pe} , a stationary state on a circular orbit as θ_{pc} , and an example trajectory as a dotted line. If the values of states are in the area denoted as circle *a*, then $\theta \simeq 0$ and

$\dot{\theta} \simeq 0$. They then have certain differences from θ_{pe} and $\dot{\theta}_{pe}$, as indicated by arrow A. In this case, the stability function V_c incorrectly evaluates the values of states to be much more stable than they actually are. On the other hand, in the area denoted as circle *b*, the values of states are close to the periodic solution ($\theta \simeq \theta_{pe}$ and $\dot{\theta} \simeq \dot{\theta}_{pe}$), which means the libration is in a stable state. However, the stability function V_c does not evaluate it as a stable state. In addition, the libration is controlled toward θ_{pc} , not θ_{pe} , as indicated by arrow B; the libration is thus destabilized through the inappropriate libration control.

As explained above, the stability function accounting for the eccentricity V_e evaluates the stability more appropriately than V_c , and the libration control toward the periodic solution θ_{pe} stabilizes the libration more efficiently than that toward θ_{pc} . The appropriate stability evaluation and the efficient libration control therefore facilitate a more rapid and secure orbital descent.

Numerical Simulation

Libration Control

To confirm the effectiveness of the libration control strategy, a numerical simulation has been conducted using $e = 0$,

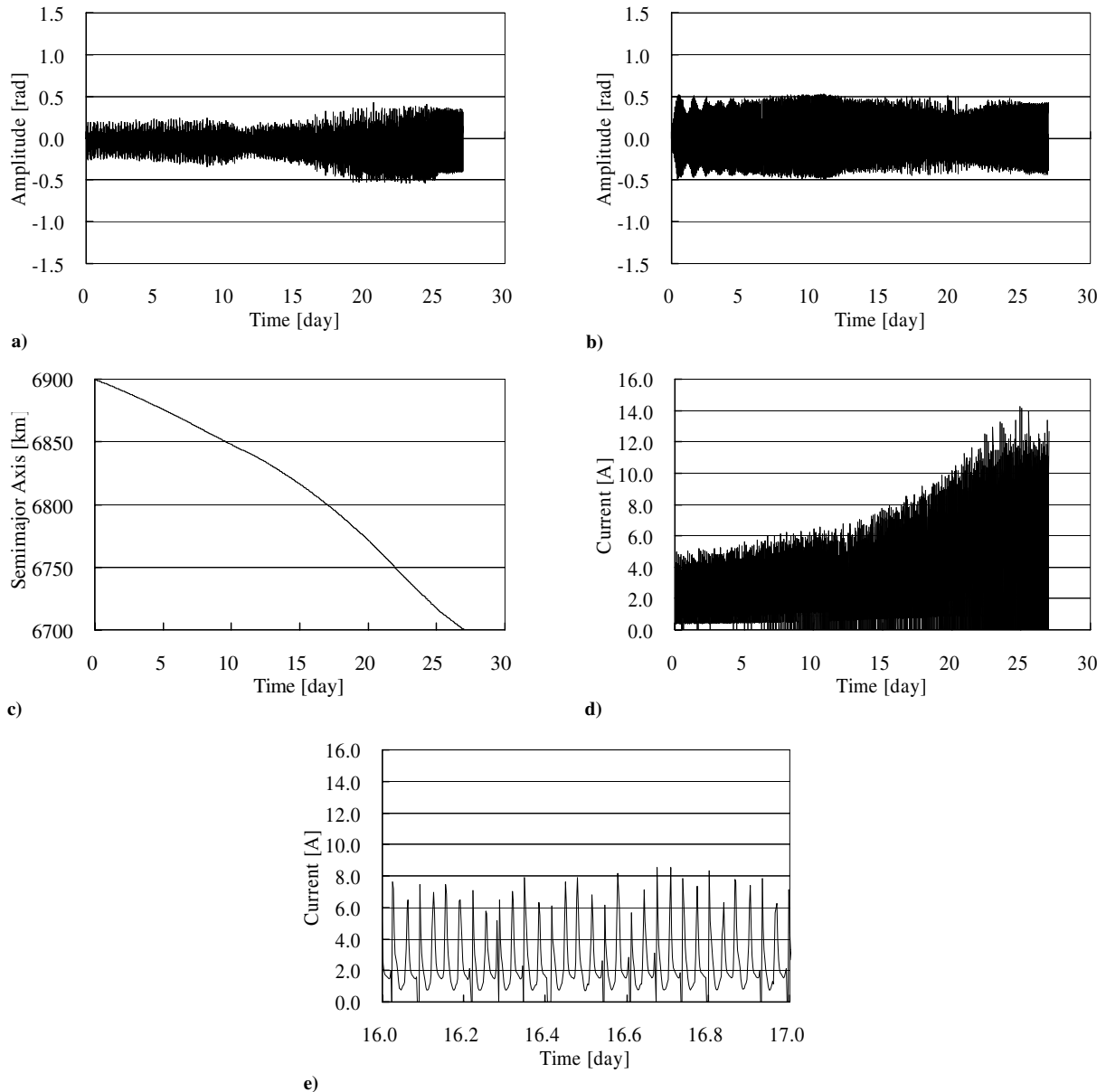


Fig. 7 Controlled motion of EDT, a) in-plane libration, b) out-of-plane libration, c) semimajor axis, d) electric current, e) close-up of electric current.

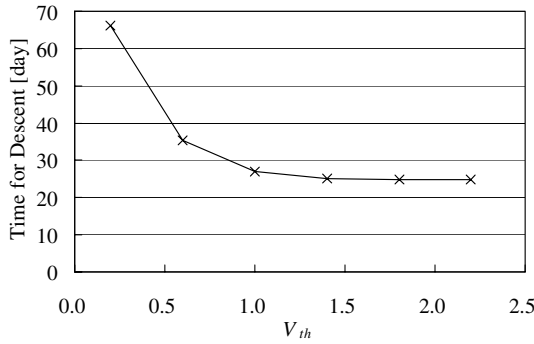


Fig. 8 Time needed for orbital descent.

$a = 6900$ km, $i = 60$ deg, $M = 3000$ kg, $m = 50$ kg, and $l = 5$ km as initial parameters. $V_{th} = 1.0$ is used for the control threshold. The simulation continued until the semimajor axis decreased to $a = 6700$ km. The result is shown in Fig. 7. The amplitudes of both in-plane and out-of-plane libration are kept within certain bounds during the orbital descent. Compared with the uncontrolled case illustrated in Fig. 3, this clearly indicates that the libration control through the current switching works as desired.

Control Threshold V_{th}

Several numerical simulations have also been conducted using the control threshold $V_{th} = 0.2 \sim 2.2$. In all cases, the EDT maintained stable libration during its orbital descent. The time needed for the orbital descent is summarized in Fig. 8. The orbital descent is more rapid when the control threshold is larger. However, the rapidity of descent seems to have an upper limit because it becomes only slightly more rapid beyond a certain value ($V_{th} = 1.0$ in this case). The cases of $V_{th} = 0.6$ and $V_{th} = 1.0$ are studied to clearly understand how the threshold V_{th} determines the libration control, and the time-histories of the stability function are shown in Fig. 9, where \times indicates when the libration is controlled. The Lorenz force inherently destabilizes the libration over a long term, but it both stabilizes and destabilizes in a short term, such as a few orbital periods, which appears as a periodic increase and decrease in the figures. Hence, the stability function naturally has a certain amplitude. When the control threshold is below the maximum value of the natural amplitude, the libration control is applied frequently, as shown in Fig. 9a, which results in a longer time of switching off the current, and the time needed for the orbital descent increases. However, when the threshold exceeds the natural amplitude, the libration control is applied only when it is necessary to stabilize the libration, as shown in Fig. 9b, which maximizes the time the current is switched on. In such cases, the threshold level hardly affects the rapidity of orbital descent. Therefore, it is possible to achieve a rapid and secure orbital descent simultaneously by applying an appropriately large threshold. In actual EDT missions, it is possible to measure the time-history of the stability function to determine its behavior during the beginning stage, as long as the initial tether deployment is successful, and a suitable value of the threshold can be determined.

Verification on Elliptic Orbit

To verify the effectiveness of the control accounting for the eccentricity, two numerical simulations are conducted. In one simulation, stability function V_e is used, and the libration is controlled toward the periodic solution. In the other, stability function V_c is used, and the libration is controlled toward the stationary state ($\theta = \dot{\theta} = \phi = \dot{\phi} = 0$). The initial parameters in the simulations are $e = 0.05$, $a = 7200$ km, $i = 60$ deg, $M = 3000$ kg, $m = 50$ kg, $l = 5$ km, and $V_{th} = 1.8$. The simulations are conducted until the semimajor axis decreases to $a = 7000$ km. As a result, the orbital descent using V_e required about 46 days while that using V_c required about 52 days. The time-histories of in-plane amplitudes are shown in Fig. 10. The EDT maintained libration during the orbital descent in both cases. Figure 11 shows the time-histories of V_e and

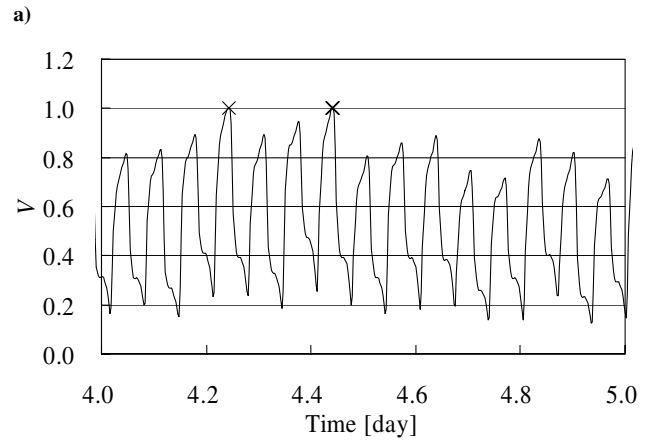
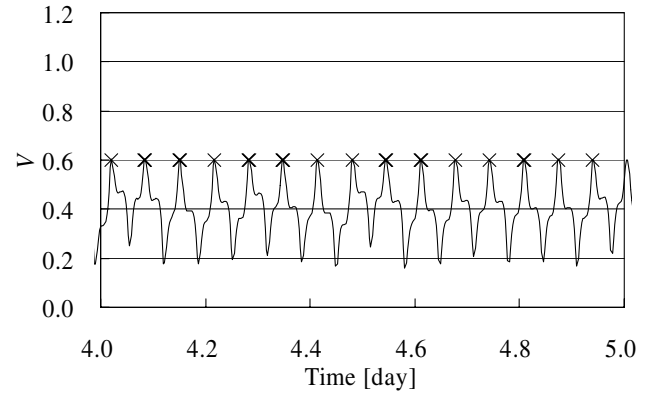


Fig. 9 Stability function, \times : libration control, a) $V_{th} = 0.6$, b) $V_{th} = 1.0$.

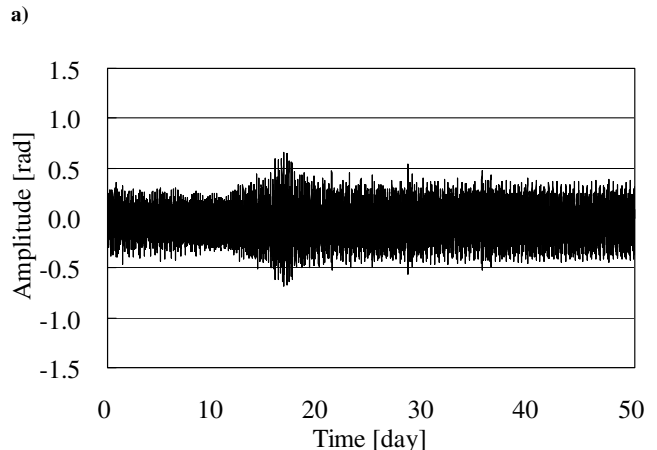
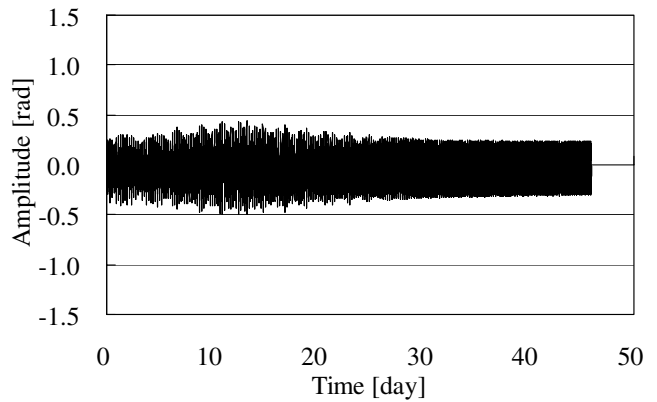


Fig. 10 In-plane amplitude, a) V_e case, b) V_c case.

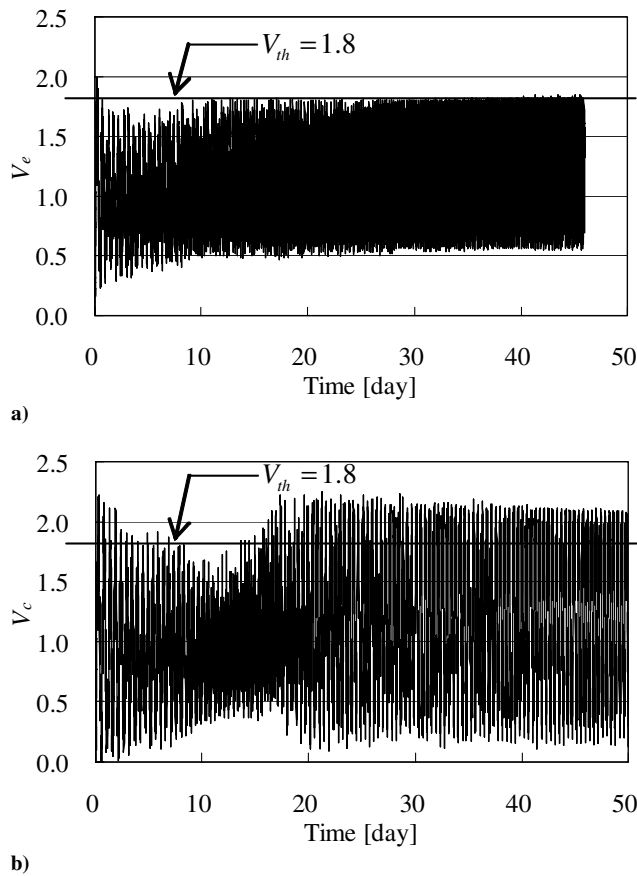


Fig. 11 Time-history of stability function, a) V_e , b) V_c .

V_c . When using V_e , the upper limit of V_e almost coincides with the threshold V_{th} , which means that the libration control toward the periodic solution works properly. However, when using V_c , the upper limit of V_c improperly exceeds V_{th} . As mentioned before, V_c evaluates the stability improperly, and the control objective is also improper so that the control cannot always stabilize the libration. These results clearly indicate that the control strategy accounting for the eccentricity facilitates a more rapid and secure orbital descent.

Conclusion

This paper has presented a practical and advanced operation strategy through current switching for deorbit of an electrodynamic tethered system (EDT). The advantages of the presented strategy are clearly demonstrated through some numerical simulations.

The in-plane direction of the Lorenz force is always opposite to the flight direction, and out-of-plane direction is easily identified from the north-south direction of the orbital motion. By taking advantage of this characteristics, both in-plane and out-of-plane libration can be simultaneously stabilized by a simple switching control, which determines only the on/off status of the current. Furthermore, the switching libration control can avoid unpredictable instability arising from the uncertainty of thermal and electrical environments. By introducing a stability function and a control strategy accounting for the orbital eccentricity, a proper stability evaluation and libration control on elliptic orbits are achieved. These advantages of the presented strategy enable a rapid and secure deorbit operation.

It is expected that the operation strategy presented in this paper will be easily materialized in actual systems. It is also expected that it will facilitate the autonomous and secure operation of an EDT on an arbitrary orbit properly accounting for the continuous changes of orbital parameters. However, there still remain some important

subjects for future work. It is expected that a stability function with a more rigorous theoretical basis enables more rapid and secure operation. It is also necessary to investigate the feasibility to cope with the switching control with other types of orbital transfers than the orbital descent. EDTs would then become a general orbital-transfer system.

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